Cournot with n firms

Assume there are n firms, all equal, competing in quantities simultaneously. Assume a linear demand: $D = A - BQ$. On the other hand, all firms have zero fixed costs and constant, equal marginal costs. Obtain the optimal quantities, the price, and explain the results.

Solution

First, we set up the profit function for a single firm j:

$$B_j = (A - BQ)q_j - c_j q_j$$

The total quantity produced in the market, Q, is the sum of the quantities produced by all the firms:

$$Q = \sum_{i=1}^{n} q_i$$

We can express the total quantity produced by all firms except firm j as Q_{-j}:

$$Q_{-j} = Q - q_j$$

To find the optimal quantity for firm j, we take the first-order condition (FOC) with respect to q_j:

$$A - BQ_{-j} - 2Bq_j - c_j = 0$$

Solving for q_j:

$$q_j = \frac{A - BQ_{-j} - c_j}{2B}$$

Since all firms are symmetric, the total quantity produced in the market is just the quantity produced by each firm multiplied by the number of firms:

$$\sum_{i=1}^{n} q_i = nq_j$$

Substituting this expression into the equation for q_j:

$$q_j = \frac{A - B(n-1)q_j - c_j}{2B}$$

Rearranging the terms:

$$q_j = \frac{A - c_j}{2B} - q_j \frac{n-1}{2}$$

$$q_j(1 + \frac{n-1}{2}) = \frac{A - c_j}{2B}$$

$$q_j(\frac{1+n}{2}) = \frac{A - c_j}{2B}$$

$$q_j = \frac{A - c_j}{B(n+1)}$$

Now, we can find the market price as a function of total quantity produced:

$$P(Q) = A - B(n\frac{A - c_j}{B(n+1)})$$

$$P(Q) = A - (n\frac{A - c_j}{(n+1)})$$

As the number of firms in the market approaches infinity, the market price converges to the marginal cost:

$$\lim_{n \to \infty} A - (n\frac{A - c_j}{(n+1)}) = A - A + c_j = c_j$$

This result shows that, in the Cournot competition model with a large number of firms, the market price converges to the marginal cost as the number of firms increases. This can be explained as follows:

As the number of firms in the market increases, competition among the firms becomes more intense. Each firm, in an attempt to maximize its profit, produces more output, which leads to an increase in the total quantity supplied in the market. This increase in quantity supplied drives the market price down.

In the limit, as the number of firms approaches infinity, the competition among the firms becomes so intense that each firm is forced to set its price equal to its marginal cost to remain in the market. This is because any price above the marginal cost would be undercut by competitors, and any price below the marginal cost would lead to a loss for the firm. Therefore, the price converges to the marginal cost in the limit.

This result is consistent with the idea that competition leads to more efficient outcomes. In this way, the Cournot competition model with a large number of firms approximates the result of perfect competition, where the market price is equal to the marginal cost.